The Geometric Hypothesis of the Meno of Plato

(L'HYPOTHÈSE GÉOMÉTRIQUE DU MÉNON DE PLATON**)**

By Paul Tannery (1876)

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This translation presents, for the first time in English, Paul Tannery's 1876 essay *L'hypothèse géométrique du Ménon de Platon*, published in *Mémoires scientifiques* (vol. 3, pp. 39–45, Paris: Gauthier-Villars). Tannery, one of the most important French historians of mathematics, here attempts to clarify Plato's obscure passage on the "geometric hypothesis" in the *Meno* (*Meno* 86e–87a), situating it within the traditions of Euclidean geometry and the interpretation of Greek mathematical language. The translation aims to remain faithful to Tannery's technical terminology, preserving key Greek expressions in transliteration while rendering his nineteenth-century French into clear English, thereby making accessible a critical investigation into the origins of geometry.

I. Apart from the passage on the "nuptial number" in Book VIII of the *Republic*[1], there is hardly any mathematical locus in Plato that has not, until now, received a truly satisfactory explanation. There is one such passage in the Meno[2].

To explain a mode of reasoning he employs, Socrates shows how geometers use it:

"When I say: — to examine according to a hypothesis — I mean this method commonly used by geometers. If they are questioned about a *figure* (chōriou), for example, if they are asked whether this triangular figure here can be inscribed in this circle here (whether this triangular figure can be inscribed in this circle), they will answer this: — I do not yet know how it stands, but I think that for this question it is appropriate to adopt a hypothesis of the following kind."

Here follows the obscure passage, which reads:

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[1] See the *Revue philosophique* février 1876. (Voir plus haut, n 2.]

[2] Platon, édition Didot, vol. I, p. 454, 1· 52; p. 455, 1. 4· [86 e-87 a.]

"If this figure is of such a kind that, when a line is drawn alongside the given side, it falls short of a figure of such a kind as it would be if it were extended..."

(εἰ μὲν ἐστὶν τοῦτο τὸ χωρίον τοιοῦτον οἷον παρὰ τὴν δοθεῖσαν αὐτοῦ γραμμὴν παρατείναντα ἐλλείπειν τοιούτῳ χωρίῳ οἷον ἂν αὐτὸ τὸ παρατεταμένον ᾖ —)

matches critical editions of Plato's *Meno* (86e–87a).

— and in which all commentators agree, they recognize the condition under which the triangle may be inscribed in the circle.

Socrates continues:

"If that is so, then such-and-such a result will follow; and something else will follow if this condition is not fulfilled. Once this hypothesis is established, I will agree to answer you regarding what results from it for the inscription of this figure in the circle, and to say whether this inscription is possible or not."

II. It is not, as in the case of the passage from the *Republic*, a lack of complete explanations that we encounter here. Those that have been offered could almost make up a whole volume [3]— but all those that deserve consideration, as far as we know, run into serious difficulties.

Unable to relate the text to the general case of a triangle to be inscribed in a given circle, they suppose instead that it concerns a triangle that is either right-angled, or isosceles, or even both right-angled and isosceles. In fact, Socrates previously drew several triangles of this last type for another purpose. He would therefore be referring to one of them (tonde), as well as to a circle (tonde) that he would be sketching while continuing his discourse.

It is already surprising that Plato should omit — precisely when he sets out to explain what a geometric hypothesis is — an essential condition of the example he takes. But even if one overlooks this, it is easy to see that, for the inscription to be possible, it is both necessary and sufficient that the hypotenuse of the right triangle be equal to the diameter of the circle.

Moreover, since in ancient geometry a circle is never given without its diameter being known, it becomes entirely incomprehensible that, instead of stating such a simple condition — which would completely satisfy the explanatory aim — Socrates should transform it into something much more complicated. He does not need to dazzle Meno with his knowledge; above all, he must strive to be understood[4].

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[3] See, in Cousin's translation, a very long note. — In his 'philological dissertation' of *Platone mathematicus* (Bonn, 1861), G. Blass acknowledges twenty-seven attempts. Let us note in passing that this 'philologist' understands nothing of mathematics. [Cf. further on, vol. II, no. 45.]

[4] Our conclusions in this respect are those of the learned and much-lamented Hankel (*Zur Geschichte der Mathematik*... Leipzig, 1874, p. 134)

III. This twofold difficulty cannot be resolved if one holds on to the assumption common to the most widely accepted explanations, namely, the identification of the term parateinein with the expression paraballein, which is the classical usage in Euclid.

To apply (paraballein) the area A along the line p (paraballein to chōrion A para tēn grammēn p) means: to construct, on a straight line of length p, a rectangle whose area is **A** — which amounts to finding another straight line **x** such that $\mathbf{p \cdot x = A}$**, or** $\mathbf{x = A/p}$. Arithmetically, then, *paraballein* para will mean: to divide by (cf. Diophantus).

In truth, it does appear that in *Book VII* of the *Republic* (Plato, Didot ed., II, p. 182, line 48), *parateinein* is used in the classical sense of *paraballein* as it appears in Euclid. But we also know that Plato's mathematical language is not so fixed that this similarity should be considered of decisive importance[5].

What has most captivated commentators is that identifying parateinein with an expression from Euclid allows one to explain *elleipein* in an analogous manner.

Indeed, the simple *application* (parabolē) explained above can also be performed in a more complex way (Euclid, Elements VI.28–29), that is, either with *defect* or with *excess* (hōste elleipein ē hyperballein) of a rectangle similar to a given rectangle.

In this case, the two rectangles (the one applied and the one in defect or in excess) have the same height, which is the unknown to be constructed; the sum or the difference of their bases is the given line[6].

Let **A** be the area to be applied (here, the area of the given triangle), **P** the given line, the parameter (here, the base of the triangle, along the given line, *para tēn dotheisan autou grammēn*), **m** the given ratio of base to height of the defective rectangle (elleiptikou), **x** the common height.

Then the equation that translates the Euclidean construction analytically in the case of defect (elleipsis) is[7]:

$$\mathbf{p \cdot x - m \cdot x^2 = A}$$

If we want to apply this language to the interpretation of Plato's passage, we must first suppose that the defective figure is a square; that is, $m = 1$.

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[5] See the *Revue philosophique* of February 1876, p. 185, the note on δύναμις used in the opposite senses of 'root' and of 'square.' [Above, p. 33, n. 2.]

[6] The problem comes down to constructing geometrically a root of a complete quadratic equation; moreover, it is from the terms used in this context that the names parabola, ellipse, and hyperbola — later given to the conic sections — were derived.

[7] In the case of Γυπερβολή (*Hyperbola*), it would be: $px - mx^2 = A$.

If we denote by h the height of the triangle dropped onto base p, and note that the area of the triangle $A = (1/2) \cdot p \cdot h$, then the condition stated by Plato — that the defective figure equals A — becomes:

$$\mathbf{x = h = p / 2,}$$

That is to say, the height of the given triangle is equal to half of its base.

It is clear that this condition is neither necessary nor sufficient for the inscription of the triangle in a given circle. There must therefore be an unstated condition relative to the circle.

The most straightforward approach is to draw it on the base as a diameter, but this is not sufficient. One must also assume that the triangle is either isosceles or right-angled. From either of these conditions, together with the one expressed, the other can be deduced.

It is difficult to believe that Plato would have assumed the triangle to be isosceles without stating so. But if the triangle is right-angled and the circle is drawn on the base as a diameter, the inscription is always possible, and the sole condition expressed in such a diffuse manner becomes superfluous.

The only remaining supposition is that Plato considered the triangle as given only in area, not in type, and that the problem is in fact to inscribe in a circle drawn on the base as diameter a triangle for which, in addition to the base, the height is known. Then the condition he expresses would be that of the maximum height it can reach, beyond which the problem is no longer possible. If this is the true meaning he attached to his words, it must be admitted that he expressed himself in a manner as defective as it is obscure.

IV. We claim, for our part, only to propose a conjecture. It, too, admittedly, suffers from a real difficulty, but it has at least the advantage of assuming perfect simplicity in Socrates' language.

We will assume that he is speaking of an arbitrary, fully given triangle, and of a circle, also given, which he draws concurrently on the sand. Wishing to inscribe the triangle in the circle, he begins by inscribing one of its sides in it, τὴν δοθεῖσαν αὐτοῦ γραμμήν (*tēn dotheisan autou grammēn)*, which we shall call the base.

This inscription is always possible, provided that this base is less than the diameter — an obvious condition a priori, and one which it was indeed unnecessary to express.

That established, the necessary and sufficient condition for the inscription is that the angle at the vertex of the triangle opposite the base is equal to one of those measured by half of one of the intercepted arcs — in particular, equal to the angle formed by the base inscribed with the circumference of the circle, which Euclid calls the angle of the segment[8].

One does not need to be very advanced in geometry to recognize that this condition is the first that presents itself to the mind. It is, moreover, the one that would immediately be drawn from the text, if one could give χωρίον (chōrion) the sense of "angle," or if one read γωνΐδιον (gōnídion), for example.

It is enough to translate literally:

"If this angle (Socrates clearly shows it on the drawn figure, and his gestures correct the vagueness of his language) is such that, beside the given line by which it is subtended, there is lacking (to reach the circumference of the circle) an angle precisely such that the subtended itself...[9]"

V. We will not propose, instead of *chōrion,* a reading of a doubtful Hellenism, where *touto to chōrion* clearly refers to the *tode to chōrion trigōnon* that precedes it.

One must therefore admit, in Plato's language, a certain confusion between the notion of angle and that of triangle; or, in other words, that he will have spoken according to the faulty (vicious) definition: "An angle is the space enclosed between two intersecting lines."

But imprecise as Plato's mathematical language may have been, could he have employed an expression that was entirely *atopic* to the time of Euclid?

We would answer that it is precisely by studying the latter author that the idea of the above conjecture came to us.

Indeed, one finds in his language and demonstrations the unmistakable traces of earlier and different geometric habits. An angle is consistently defined, for example, in a triangle, by means of the subtending line or base, and the two sides, whereas today we hardly apply these definitions except to the triangle itself. There was, at a particular time, real confusion between the terms used to designate the angle at the vertex of a triangle and the triangle itself. The text of Plato would provide a topical proof of this confusion.

In summary, we would propose translating the statement of the hypothesis as follows:

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[8] We would rather say today: the angle of the inscribed base with the tangent at the extremity.

[9] *Παρατείνειν* [parateinein] would be taken in the classical sense of *ὑποτείνειν* [hypoteinein], whence (i) *ὑποτείνουσα τὴν ὀρθὴν γωνίαν* [the line which subtends the right angle], said constantly by Euclid for the hypotenuse of a right-angled triangle (cf. *Elements* I.17, I.47, I.48). The substitution of one prefix for the other is entirely natural in Plato, if the line taken as the base was at the side and not at the bottom of the figure. As for the particular active sense of *παρατείναντα* [parateinanta], it refers without difficulty to the construction that still remains to be carried out for the inscription.

"If this triangle is such that the angle formed by the given base (and the circumference of the circle) is precisely equal to that subtended by this base, then such-and-such will result..."

This interpretation would allow us to suppose that Plato intended to allude to the recent discovery of an important theorem — the equality of all angles inscribed in the same segment of a circle — which is historically known to have been unknown to a barely earlier geometer, Hippocrates of Chios.

Thus, in the *Theaetetus*, he has us witness the generalization of the notion of the incommensurable roots of non-square perfect numbers.

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